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Edge Envelope Equation for a Ballistically Focused Neutralized Ion Beam

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EDGE ENVELOPE EQUATION FOR A BALLISTICALLY FOCUSED NEUTRALIZED ION BEAM

by

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ABSTRACT

An envelope equation for a cold ion beam with overall charge and current neutralization provided by a coflowing electron gas obeying an adiabatic equation of state is derived. The derivation assumes the beam evolves self-similarly with the ion at the edge of a uniform density ion profile. Numerical and approximate analytical solutions are calculated.

Ballistic focusing of intense beams of energetic ions into a small focal spot size is required for some confinement fusion schemes.[1,2] When the ions are cold, beam self electrostatic fields limit its focusing ability.[3] This limit can be relaxed by mixing an equal number of co-moving electrons with the ions. However, unless the electrons are also cold, their neutralizing effect is incomplete over a Debye sheath at the beam edge.

In order to determine the effect of fields originating in the Debye sheath upon the evolution of a neutralized ion beam, we have developed an envelope equation for an axially symmetric, uniform density, beam of cold ions, charge neutralized with a comoving, two-dimensional electron gas obeying an adiabatic equation of state. An important physical feature of this model is that the electron Debye length is invariant to changes in beam radius. Therefore, as the radius of a focused beam decreases, opposing electrostatic forces become increasingly stronger and eventually arrest the focus.

An envelope equation for the root mean square (RMS) radius has been derived and investigated elsewhere.[4] Here we derive an envelope equation by assuming the beam evolves self-similarly with an ion at the edge of a uniform density ion profile. The ion at the beam edge feels the effect of more space charge than any other ion. The edge envelope equation will, therefore, predict larger focal spot sizes than the RMS envelope equation.

The edge, like the RMS envelope equation, is based on the radial equation of motion for a beam ion of axial velocity V_z in a radial electric field, $E_r(r)$, arising from incomplete local charge neutralization.

$$\ddot{r} = \frac{Z_i e E_r(r)}{M_i} \quad . \tag{1}$$

When there are no crossings of ion trajectories, Eq. (1) for an ion at the beam edge where r = R, can be converted to an envelope equation by applying the transformation $R = V_z R'$ and making the paraxial approximation V_z = constant, with the result

$$R'' = \frac{Z_i e E_r(R)}{M_i} \quad . \tag{2}$$

Here the prime indicates a derivative with respect to the axial coordinate z. Self magnetic fields and other relativistic effects may be included in the paraxial approximation by substituting the relativistic transverse mass, $\gamma^3 M_i$, for M_i in Eq. (2), where $\gamma = (1-V_z^2/c^2)^{-\frac{1}{2}}$. This model has been used previously to calculate electron[5], as well as ion[6] beam envelopes by specifying a uniform beam charge distribution and determining $E_r(R)$ from it.

In the present work, $E_r(R)$ is determined from a self-consistent model of the ion and electron radial density profiles, $n_i(r)$ and $n_e(r)$, through Poisson's equation

$$\frac{1}{r}\frac{d}{dr}(rE_r) = 4\pi e(Z_i n_i - n_e) \quad . \tag{3}$$

The ion density profile is uniform out to the beam edge, then vanishes so that

$$n_i(r) = \frac{N_L}{\pi R^2} \; H(R - r) \quad , \tag{4}$$

where N_L is a line density and H(x) is the Heaviside function. The electron density profile is determined from steady state fluid equations. These are the radial force balance equation

$$\frac{\partial P_e}{\partial r} = -n_e e\, E_r \quad , \tag{5}$$

where $P_e(r)$ is the electron pressure profile and an adiabatic equation of state for a gas with two degrees of freedom

$$P_e/n_e^2 = \text{constant.} \tag{6}$$

Equations (5) and (6) are valid for the present application as long as electron heating is two-dimensional and changes in the beam envelope occur slowly compared to the electron thermal velocity; i.e., $V_z R' << (P_e/M_e n_e)^{\frac{1}{2}}$. Finally, $n_e(r)$ is normalized to provide for overall charge neutralization

$$Z_i N_L = 2\pi Z_i \int_0^\infty dr\; r\; n_i = 2\pi \int_0^\infty dr\; r\; n_e \quad . \tag{7}$$

Equations (3)-(7) are easily solved for $n_e(r)$, $P_e(r)$, or $E_r(r)$. Specifically, using Eqs. (5) and (6) to eliminate $E_r(r)$ and $P_e(r)$ from Eq. (4) results in

$$\frac{1}{r} \frac{\partial}{\partial r}\, r \frac{\partial}{\partial r}\, n_e = \frac{(n_e - Z_i n_i)}{2\lambda_D^2} \quad , \tag{8}$$

where the Debye length, λ_D, defined as $\lambda_D^2 = P_e/4\pi e^2 n_e{}^2$, is constant with respect to changes in both r and R. Using Eq. (4), Eq. (8) is solved in terms of combinations of zero order modified Bessel functions of the first and second kinds, $I_0(x)$ and $K_0(x)$, in the regions for which $r<R$ and $r>R$. When continuity and smoothness in $n_e(r)$ are required, the solution is given explicitly by

$$n_e(r) = \frac{N_L}{\pi R^2} \quad 1 - \frac{R}{\lambda_D\sqrt{2}} K_1\left(\frac{R}{\lambda_D\sqrt{2}}\right) I_o \left(\frac{r}{\lambda_D\sqrt{2}}\right) \quad \text{for } r < R$$

and (9)

$$n_e(r) = \frac{N_L}{\pi R^2} \frac{R}{\lambda_D\sqrt{2}} I_1\left(\frac{R}{\lambda_D\sqrt{2}}\right) K_o \left(\frac{r}{\lambda_D\sqrt{2}}\right) \text{for } r > R \quad .$$

From Eqs. (4) and (9), it is evident that the dimensionless parameter $R/\lambda_D\sqrt{2}$ determines the degree of local charge neutralization. For instance, neutralization is good when $R/\lambda_D\sqrt{2} >> 1$ and relatively poor when $R/\lambda_D\sqrt{2} \lesssim 1$. These facts are illustrated in Fig. 1.

Derivation of the ion envelope equation is completed by solving Eqs. (5), (6), and (9) for $E_r(r)$, evaluating it for $r = R$, and making the substitution into Eq. (2). The result is

$$R'' = \frac{2k \; I_1(R/\lambda_D\sqrt{2}) \; K_1(R/\lambda_D\sqrt{2})}{R} \quad , \tag{10}$$

where the dimensionless parameter k is the ion beam perveance defined by

$$k = \frac{2N_L Z_i^2 e^2}{M_i V_z{}^2} \quad .$$

Equation (10) may be analytically integrated once resulting in

$$\frac{R'^2}{2k} + I_0\left(\frac{R}{\lambda_D\sqrt{2}}\right) K_0\left(\frac{R}{\lambda_D\sqrt{2}}\right) + I_1\left(\frac{R}{\lambda_D\sqrt{2}}\right) K_1\left(\frac{R}{\lambda_D\sqrt{2}}\right) = \text{constant} \quad , \tag{11}$$

which may also be derived by requiring the conservation of particle and field energy. It is noteworthy that Eq. (10) reduces to expected results in both limits $R/\lambda_D\sqrt{2} \to 0$ and $R/\lambda_D\sqrt{2} \to \infty$.

Equation (10) may be used to describe the envelope of a neutralized ion beam emerging from an ion rocket.[7] However, of present interest are solutions with boundary conditions characteristic of focused beams. In such solutions the beam envelope reaches a minimum radius, ΔR, at the point where $R' = 0$. This point is related to the boundary values through Eq. (11) by

$$\frac{\tan^2\theta}{2k} + I\,(R_o/\lambda_D\sqrt{2})\;K_o\,(R_o/\lambda_D\sqrt{2}) + I_1(R_o/\lambda_D\sqrt{2})\;K_1(R_o/\lambda_D\sqrt{2})$$

$$= I_o(\Delta R/\lambda_D\sqrt{2})\;K_o(\Delta R/\lambda_D\sqrt{2}) + I_1(\Delta R/\lambda_D\sqrt{2})\;K_1(\Delta R/\lambda_D\sqrt{2}) \qquad (12)$$

where θ, the initial focal angle, is defined by $\tan^2\theta = (R')_o^2$.

Numerical solutions to Eq. (12) over several orders of magnitude in the parameters $\tan^2\theta/2k$, $R_o/\lambda_D\sqrt{2}$, and $\Delta R/R_o$ are provided in Fig. 2. Also, approximate analytic solutions based on leading order expansions of the products of modified Bessel functions in Eq. (12) are available for all three possible orderings of R_o, ΔR, and $\lambda_D\sqrt{2}$:

1. $R_o \gg \Delta R \gg \lambda_D\sqrt{2}$, which results in the formula

$$\frac{\Delta R}{R_o} = 1 + \frac{Ro}{\lambda_D\sqrt{2}}\,\frac{(\tan^2\theta)^{-1}}{2k} \qquad (13)$$

and consequently is valid in the parameter regime

$$1 \gg \tan^2\theta/2k \gg \lambda_D\sqrt{2}/R_o,$$

2. $R_o \gg \lambda_D\sqrt{2} \gg \Delta R$ which results in

$$\frac{\Delta R}{R_o} = \frac{\lambda_D\sqrt{2}}{R_o}\exp\left(\frac{1}{2} - \frac{\tan^2\theta}{2k} - \frac{\lambda_D\sqrt{2}}{R_o}\right) \qquad (14)$$

and is valid for $\tan^2\theta/2k \gg 1 \gg \lambda_D\sqrt{2}/R_o$ and

3. $\lambda_D\sqrt{2} \gg R_o \gg \Delta R$ resulting in

$$\frac{\Delta R}{R_o} = \exp\,(-\tan^2\theta/2k) \qquad (15)$$

and requiring only $\lambda_D\sqrt{2}/R_o \gg 1$.

Equations (13) and (14) are new, while Eq. (15) gives the well known spot size limit for unneutralized charge particle beams.[3] Comparisons with numerical results indicate Eqs. (13)-(15) give values of $\Delta R/R_0$, which differ from values determined from Eq. (12) by 50% at most.

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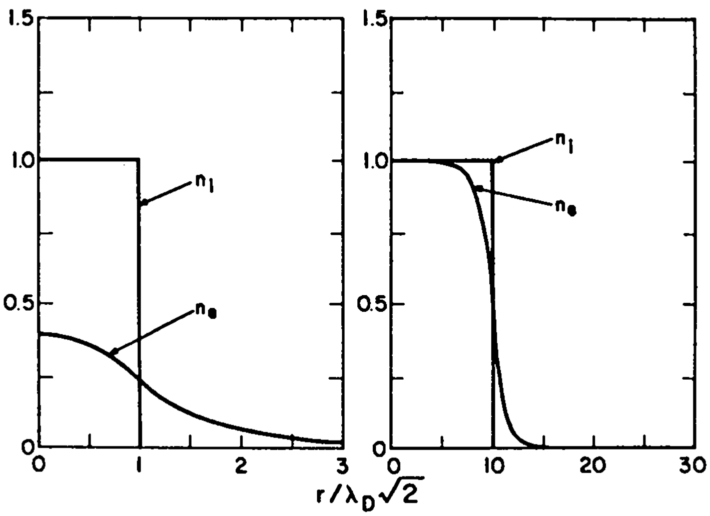

Fig. 1

Beam ion and electron density profiles, $n_i(r)$ and $n_e(r)$, for a) $R/\lambda_D\sqrt{2} = 1$ and b) $R/\lambda_D\sqrt{2} = 10.0$ where R indicates beam edge radius.

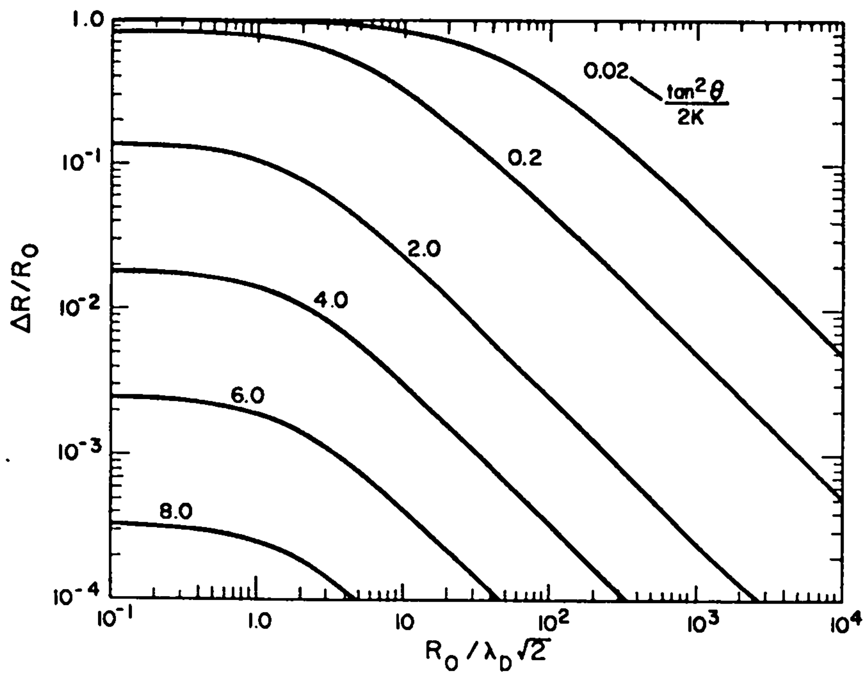

Fig. 2

Solutions to edge focal spot equation, (12), in terms of dimensionless parameters $\Delta R/R_0$, $R_0/\lambda_D\sqrt{2}$, and $\tan^2\theta/2K$. ΔR is the focused beam radius, R_0 is the initial beam radius, λ_D is the electron Debye length, θ is the beam focal angle, and K is the ion beam perveance.

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